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On a stochastic fractional partial differential equation driven by a Lévy space-time white noise ☆

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ABSTRACT

We study the existence and uniqueness of the global mild solution for a stochastic fractional partial differential equation driven by a Lévy space-time white noise. Moreover, the flow property for the solution is also studied.

1. Introduction

Let $0 < \lambda \leqslant 2$, and Δ_λ be the λ-fractional differential operator, which is defined via Fourier transform $\Im$ by

$$\Im(\Delta_\lambda u)(\xi) = -|\xi|^\lambda \Im(u)(\xi), \quad u \in D(\Delta_\lambda),\ \xi \in \mathbb{R}.$$

In this article, we consider a stochastic partial differential equation (abbr. SPDE) with Lévy space-time white noise, which is induced by the λ-fractional differential operator. Formally, we write the SPDE as follows:

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} = \Delta_\lambda u(t,x) + f\big(t,x,u(t,x)\big) + \sigma\big(t,x,u(t,x)\big)\dot{L}(x,t), \\ u(0,x) = u_0(x), \end{cases} \tag{1.1}$$

where $(t,x) \in [0,\infty) \times \mathbb{R}$, and $\dot{L}(x,t)$ is a Lévy space-time white noise, which consists of a Brownian sheet and a Poisson space-time white noise on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ (see Section 2 for precise definition). Actually, we understand this equation in Walsh [23] sense, and so we can rewrite Eq. (1.1) as follows:

$$\begin{aligned} u(t,x) = {} & \int_{\mathbb{R}} G(t,x-y)u_0(y)\,dy + \int_0^t \int_{\mathbb{R}} G_\lambda(t-s,x-y) f\big(s,y,u(s,y)\big)\,dy\,ds \\ & + \int_0^t \int_{\mathbb{R}} G_\lambda(t-s,x-y)\sigma\big(s,y,u(s,y)\big)\dot{L}(y,s)\,dy\,ds. \end{aligned} \tag{1.2}$$

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In recent years, fractional equations have received more attentions. They have been applied to model various phenomena in image analysis, risk management and statistical mechanics (see e.g. [4,5,7,11,17–19,24,25]). It seems more significant to investigate fractional equation with some random force, for example, (Gaussian) white noise as the random force perturbation or with random initial data. There have been some works involving this subject (see e.g. [2,3,6,9,10,19,15,22]). In Azerad and Mellouk [3] and Debbi and Dozzi [9], the authors discussed the solutions to a 1-dimensional stochastic fractional differential equation driven by space-time white noise. In this paper, we use a Lévy space-time white noise to model the random force. This is a natural generalization from continuous paths to right-continuous with left limit paths for stochastic dynamics. In Albeverio et al. [1] and Truman and Wu [21], the authors established the existence and uniqueness of the solutions for a class of stochastic heat equations driven by compensated Poisson random measures and Lévy space-time white noises in L^2-sense, respectively. In [20], Mueller obtained a minimal short-time solution for a stochastic heat equation with an $\alpha \in]0,1[$-stable noise, which is a different approach from [1,21]. We notice that fixed point principle and Picard iteration scheme work in [1,21], since Burkhölder–Davis–Gundy (abbr. B-D-G) inequality can be applied to estimate stochastic integration w.r.t. compensated Poisson random measures in L^2-sense. Unfortunately, the usual B-D-G inequality cannot work for estimating stochastic integration w.r.t. compensated Poisson random measures in L^p $(p > 2)$ sense. Hence a new version of B-D-G inequality (see, e.g. [8,14,16]) will be adopted for the L^p $(p \geqslant 2)$-estimates on the solution to Eq. (1.1).

The rest of the paper is organized as follows: In Section 2, we give the definition of Lévy space-time white noise, some properties of fractional Green kernel and a generalized B-D-G inequality. Section 3 is devoted to proving the existence and uniqueness of the mild solution to Eq. (1.1) in L^p $(p \geqslant 2)$ sense under some appropriate conditions. Finally, a flow property of the solution will be considered in Section 4.

Throughout the paper, the generic positive constants C may be different from line to line. If it is essential, the dependence of a constant C on some parameters, say p, will be written by C_p.

2. Preliminaries

In this section, we state the definition of Lévy space-time white noise, a generalized B-D-G inequality and some properties of fractional Green kernel.

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geqslant 0}, \mathbb{P})$ be a probability space with filtration $(\mathcal{F}_t)_{t\geqslant 0}$ admitting the usual condition and $(\mathrm{E}_i, \mathcal{E}_i, \mu_i)$ $(i = 1, 2)$ be two σ-finite measurable spaces. We call $N : (\mathrm{E}_1, \mathcal{E}_1, \mu_1) \times (\mathrm{E}_2, \mathcal{E}_2, \mu_2) \times (\Omega, \mathcal{F}, \mathbb{P}) \to \mathbb{N} \cup \{0\} \cup \{\infty\}$ a Poisson noise on $(\mathrm{E}_1, \mathcal{E}_1, \mu_1)$, if for all $A \in \mathcal{E}_1$, $B \in \mathcal{E}_2$ and $n \in \mathbb{N} \cup \{0\} \cup \{\infty\}$,

$$\mathbb{P}\big(N(A,B)=n\big) = \frac{e^{-\mu_1(A)\mu_2(B)}[\mu_1(A)\mu_2(B)]^n}{n!}. \tag{2.1}$$

In particular, if $(\mathrm{E}_1, \mathcal{E}_1, \mu_1) = ([0,\infty[\times \mathbb{R}, \mathcal{B}([0,\infty[\times \mathbb{R}), \mathrm{d}t \times \mathrm{d}x)$, then define a compensated random martingale measure by

$$M(B, A, t) = N\big([0,t] \times A, B\big) - \mu_1\big([0,t] \times A\big)\mu_2(B), \tag{2.2}$$

by assuming that $\mu_1([0,t] \times A)\mu_2(B) < \infty$ for all $(t, A, B) \in [0,\infty) \times \mathcal{B}(\mathbb{R}) \times \mathcal{E}_2$. Further, let $\phi : \mathrm{E}_1 \times \mathrm{E}_2 \times \Omega \to \mathbb{R}$ be a $(\mathcal{F}_t)_{t\geqslant 0}$-predictable function satisfying

$$\mathbb{E}\Bigg[\int_0^t \int_A \int_B \big|\phi(s,x,y)\big|^2 \mu_2(\mathrm{d}y)\,\mathrm{d}x\,\mathrm{d}s\Bigg] < \infty, \tag{2.3}$$

for all $t > 0$ and $(A, B) \in \mathcal{E}_1 \times \mathcal{E}_2$. We can define a stochastic integral process

$$R_t = \int_0^{t+} \int_A \int_B \phi(s,x,y)\, M(\mathrm{d}y, \mathrm{d}x, \mathrm{d}s)$$

which is a square integrable $(\mathbb{P}, (\mathcal{F}_t)_{t\geqslant 0})$-martingale. It is well known that, a (pure jump) Lévy space-time white noise possesses the following structure:

$$\dot{L}(x,t) = \dot{W}(t,x) + \int_{U_0} h_1(t,x,y)\, \dot{M}(\mathrm{d}y,x,t) + \int_{\mathrm{E}_2\setminus U_0} h_2(t,x,y)\, \dot{N}(\mathrm{d}y,x,t), \tag{2.4}$$

for some $U_0 \in \mathcal{E}_2$ such that $\mu_2(\mathrm{E}_2 \setminus U_0) < \infty$ and $\int_{U_0} z^2 \mu_2(\mathrm{d}z) < +\infty$. Here $h_1, h_2 : [0,\infty[\times D \times \mathrm{E}_2 \to \mathbb{R}$ are some measurable functions, and $\dot{W}(t,x)$ is a Gaussian space-time white noise on $[0,T] \times \mathbb{R}$, $\dot{M}$, $\dot{N}$ are the Radon–Nikodym derivatives defined by

$$\dot{M}(\mathrm{d}y,x,t) = \frac{M(\mathrm{d}y,\mathrm{d}x,\mathrm{d}t)}{\mathrm{d}t \times \mathrm{d}x}, \qquad \dot{N}(\mathrm{d}y,x,t) := \frac{N(\mathrm{d}t \times \mathrm{d}x, \mathrm{d}y)}{\mathrm{d}t \times \mathrm{d}x}, \tag{2.5}$$

with $(t,x,y) \in [0,\infty[\times \mathbb{R} \times \mathrm{E}_2$. Next we quote the following B-D-G inequality (see e.g. [8] or [16]):

Proposition 2.1. *Let $\phi : [0, \infty[\times \mathbb{R} \times \mathrm{E}_2 \times \Omega \to \mathbb{R}$ be $(\mathcal{F}_t)_{t \geqslant 0}$-predictable and satisfies* (2.3)*. Define the integral process by*

$$\left\{ X_t = \int_0^{t+} \int_{\mathbb{R}} \int_{\mathrm{E}_2} \phi(s, y, z)\, M(\mathrm{d}z, \mathrm{d}y, \mathrm{d}s),\ t \geqslant 0 \right\}.$$

Then for any $T > 0$ and $p > 1$, there exists a constant $C_{p,T} > 0$ such that

$$\sup_{0 \leqslant t \leqslant T} \mathbb{E}\big[|X_t|^p\big] \leqslant C_{p,T} \left[\int_0^T \int_{\mathbb{R}} \int_{\mathrm{E}_2} \big(\mathbb{E}\big[\big|\phi(s, y, z)\big|^p\big]\big)^{\frac{2}{p}} \mu_2(\mathrm{d}z)\, \mathrm{d}y\, \mathrm{d}s \right]^{p/2}. \tag{2.6}$$

Let the Green kennel $G_\lambda(t, x)$ be the fundamental solution of the Cauchy problem:

$$\begin{cases} \dfrac{\partial}{\partial t} G_\lambda(t, x) = \Delta_\lambda G_\lambda(t, x), & \text{for } (t, x) \in [0, \infty[\times \mathbb{R}, \\ G_\lambda(0, x) = \delta_0(x), \end{cases}$$

where δ_0 denotes the Dirac function. By Fourier transform,

$$G_\lambda(t, x) = \Im^{-1}\big(e^{-t|\cdot|^\lambda}\big)(x) = \int_{\mathbb{R}} e^{2i\pi x\xi} e^{-t|\xi|^\lambda}\, \mathrm{d}\xi = \Im\big(e^{-t|\cdot|^\lambda}\big)(x). \tag{2.7}$$

Let's us recall some well-known properties about $G_\lambda(t, x)$ (see, e.g. [3,9,10,15]):

Lemma 2.1. *Let $\lambda \in]0, 2]$. Then function $G_\lambda(t, x)$ is the transition density of a Lévy stable processes and satisfies*:

(a) *For all $(t, x) \in]0, \infty[\times \mathbb{R}$,*

$$G_\lambda(t, x) \geqslant 0, \quad \textit{and} \quad \int_{\mathbb{R}} G_\lambda(t, x)\, \mathrm{d}x = 1.$$

(b) *For all $(t, x) \in]0, \infty[\times \mathbb{R}$,*

$$G_\lambda(t, x) = t^{-1/\lambda} G_\lambda\big(1, t^{-1/\lambda} x\big).$$

(c) *Let $m \in \mathbb{N} \cup \{0\}$. Then there exists $C_m > 0$ such that for all $(t, x) \in]0, \infty[\times \mathbb{R}$,*

$$\big|\partial_x^m G_\lambda(t, x)\big| \geqslant t^{-\frac{1+m}{\lambda}} \frac{C_m}{1 + t^{-2/\lambda}|x|^2}.$$

(d) *For all $s, t \in]0, \infty[$,*

$$G_\lambda(s, \cdot) * G_\lambda(t, \cdot) = G_\lambda(s + t, \cdot).$$

(e) *It holds that*

$$\int_0^T \int_{\mathbb{R}} G_\lambda^\alpha(t, x)\, \mathrm{d}x\, \mathrm{d}t < \infty \quad \Leftrightarrow \quad 1/2 < \alpha < 1 + \lambda.$$

Remark 2.1. Throughout the paper, we restrict $\lambda \in]1, 2]$.

3. Existence and uniqueness

In this section, we shall prove the existence and uniqueness of the global mild solution to Eq. (1.1). Recall (1.2) and (2.4). Then for all $(t, x) \in [0, T] \times \mathbb{R}$,

$$u(t, x) = \int_{\mathbb{R}} G(t, x - y) u_0(y)\, \mathrm{d}y + \int_0^t \int_{\mathbb{R}} G_\lambda(t - s, x - y) f\big(s, y, u(s, y)\big)\, \mathrm{d}y\, \mathrm{d}s$$

$$
\begin{aligned}
&+\int_0^t\int_{\mathbb{R}} G_\lambda(t-s,x-y)\sigma\big(s,y,u(s,y)\big)\,W(\mathrm{d}y,\mathrm{d}s)\\
&+\int_0^t\int_{\mathbb{R}} G_\lambda(t-s,x-y)\sigma\big(s,y,u(s,y)\big)\psi(s,y)\,\mathrm{d}y\,\mathrm{d}s\\
&+\int_0^{t+}\int_{\mathbb{R}}\int_{\mathrm{E}_2} G_\lambda(t-s,x-y)\sigma\big(s,y,u(s,y)\big)h(s,y,z)\,M(\mathrm{d}z,\mathrm{d}y,\mathrm{d}s),
\end{aligned}
\tag{3.1}
$$

with the mappings

$$
\psi(t,y)=\int_{\mathrm{E}_2\setminus U_0} h_2(t,y,z)\,\mu_2(\mathrm{d}z),
$$

$$
h(t,y,z)=h_1(t,y,z)\mathbf{1}_{U_0}(z)+h_2(t,y,z)\mathbf{1}_{\mathrm{E}_2/U_0}(z).
$$

In what follows, we will show that such a mild solution indeed exists and is unique. First, we state the main result of this section.

Theorem 3.1. *Assume that the following conditions are satisfied*:

(i) *f,σ are uniformly Lipschitzian, i.e. there exists a constant $C>0$ such that for $(t,x)\in[0,T]\times\mathbb{R}$ and $u,v\in\mathbb{R}$,*

$$
\big|f(t,x,u)-f(t,x,v)\big|+\big|\sigma(t,x,u)-\sigma(t,x,v)\big|\leqslant C|u-v|.
$$

(ii) *For $p\in\,]\frac{2(\lambda+1)}{\lambda-1},\infty[$ with $\lambda\in\,]1,2]$,*

$$
\sup_{0\leqslant t\leqslant T}\big\|\psi(t,\cdot)\big\|_p^p<\infty,
\tag{3.2}
$$

$$
\sup_{0\leqslant t\leqslant T}\left\|\int_{\mathrm{E}_2}\big|h(t,\cdot,z)\big|^2\,\mu_2(\mathrm{d}z)\right\|_{\frac{p}{2}}^{\frac{p}{2}}<\infty.
\tag{3.3}
$$

Then for all $\mathcal{F}_0$-measurable $u_0:\mathbb{R}\times\Omega\to\mathbb{R}$ satisfying $\mathbb{E}[\|u_0(\cdot)\|_p^p]<\infty$, there exists a unique mild solution $(u(t,x))_{(t,x)\in[0,T]\times\mathbb{R}}$ to Eq. (1.1) *and for all $p\in\,]\frac{2(\lambda+1)}{\lambda-1},\infty[$,*

$$
\sup_{0\leqslant t\leqslant T}\mathbb{E}\big[\big\|u(t,\cdot)\big\|_p^p\big]<\infty.
$$

To prove the theorem, we need the following lemma:

Lemma 3.1. *Let $p\in[1,\infty[$, $\rho\in[1,p]$ and $r\in[1,\infty[$ such that*

$$
\frac{1}{r}=\frac{1}{p}-\frac{1}{\rho}+1\in\,]0,1].
$$

Let $G_\lambda=G_\lambda(t,x-y)$ be the Green kernel, $H=G_\lambda$ or G_λ^2 with $(t,x,y)\in[0,T]\times\mathbb{R}^2$. Define an operator $\mathscr{L}$ by

$$
\mathscr{L}(v)(t,x)=\int_0^t\int_{\mathbb{R}} H(t-s,x-y)v(s,y)\,\mathrm{d}y\,\mathrm{d}s,
$$

with $v\in L^1([0,T];L^\rho(\mathbb{R}))$. Then $\mathscr{L}:L^1([0,T],L^\rho(\mathbb{R}))\to L^\infty([0,T];L^p(\mathbb{R}))$ is a bounded linear operator and satisfies that

(a) *If $H=G_\lambda$, then there exists a constant $C>0$ such that for all $r\in[1,1+\lambda[$,*

$$
\big\|\mathscr{L}(v)(t,\cdot)\big\|_p\leqslant C\int_0^t(t-s)^{-\frac{1}{\lambda}(1-r)}\big\|v(s,\cdot)\big\|_\rho\,\mathrm{d}s,\quad \forall t\in[0,T].
$$

(b) *If $H = G_\lambda^2$, then there exists a constant $C > 0$ such that for all $r \in [1, \frac{1}{2}(1+\lambda)[$,*

$$\left\|\mathscr{L}(v)(t,\cdot)\right\|_p \leqslant C\int_0^t (t-s)^{-\frac{1}{\lambda}(2-r)}\left\|v(s,\cdot)\right\|_\rho \,\mathrm{d}s, \quad \forall t\in[0,T].$$

Proof. We only prove the case (a), since the proof of (b) is similar. From Minkowski's inequality, (b) of Lemma 2.1 and Young's inequality, it follows that

$$\begin{aligned}
\left\|\mathscr{L}(v)(t,\cdot)\right\|_p &= \left\|\int_0^t\int_{\mathbb{R}} G_\lambda(t-s,\cdot-y)v(s,y)\,\mathrm{d}y\,\mathrm{d}s\right\|_p \\
&\leqslant \int_0^t \left\|\int_{\mathbb{R}} G_\lambda(t-s,\cdot-y)v(s,y)\,\mathrm{d}y\right\|_p \mathrm{d}s \\
&\leqslant C\int_0^t (t-s)^{-\frac{1}{\lambda}}\left\|\int_{\mathbb{R}} G_\lambda\big(1,(t-s)^{-\frac{1}{\lambda}}(\cdot-y)\big)\big|v(s,y)\big|\,\mathrm{d}y\right\|_p \mathrm{d}s \\
&= C\int_0^t (t-s)^{-\frac{1}{\lambda}}\left\|\big[G_\lambda\big(1,(t-s)^{-\frac{1}{\lambda}}\cdot\big)*\big|v(s,\cdot)\big|\big](\cdot)\right\|_p \mathrm{d}s \\
&\leqslant C\int_0^t (t-s)^{-\frac{1}{\lambda}}\left\|G_\lambda\big(1,(t-s)^{-\frac{1}{\lambda}}\cdot\big)\right\|_r\left\|v(s,\cdot)\right\|_\rho \mathrm{d}s \\
&\leqslant C\int_0^t (t-s)^{-\frac{1}{\lambda}(1-r)}\left\|v(s,\cdot)\right\|_\rho \mathrm{d}s,
\end{aligned} \tag{3.4}$$

where we used the fact that for $r\in[1,3[$,

$$\begin{aligned}
\left\|G_\lambda\big(1,(t-s)^{-\frac{1}{\lambda}}\cdot\big)\right\|_r &= \left[\int_{\mathbb{R}} G_\lambda^r\big(1,(t-s)^{-\frac{1}{\lambda}}y\big)\,\mathrm{d}y\right]^r \\
&\leqslant (t-s)^{\frac{r}{\lambda}}\left[\int_{\mathbb{R}} G_\lambda^r(1,y)\,\mathrm{d}y\right]^r \\
&\leqslant C(t-s)^{\frac{r}{\lambda}}.
\end{aligned} \tag{3.5}$$

Thus we complete the proof of the lemma. □

We mainly adopt fixed point principle to prove Theorem 3.1. Let $\mathcal{H}$ be the space of all $L^p(\mathbb{R})$-valued RCLL processes $(u(t,\cdot))_{0\leqslant t\leqslant T}$ with the norm

$$\|u\|_{\mathcal{H}} := \Big\{\sup_{0\leqslant t\leqslant T} e^{-\eta t}\mathbb{E}\big[\left\|u(t,\cdot)\right\|_p^p\big]\Big\}^{\frac{1}{p}}, \quad \eta>0.$$

Then under this norm, $\mathcal{H}$ is a Banach space. Further for $u\in\mathcal{H}$, let's define an operator S by

$$S(u)(t,x) = \sum_{i=0}^4 J_\lambda^i(u)(t,x), \tag{3.6}$$

where

$$J_\lambda^0(t,x) := \int_{\mathbb{R}} G(t,x-y)u_0(y)\,\mathrm{d}y,$$

$$J_\lambda^1(t,x) := \int_0^t \int_{\mathbb{R}} G_\lambda(t-s, x-y) f\big(s, y, u(s,y)\big)\,\mathrm{d}y\,\mathrm{d}s,$$

$$J_\lambda^3(t,x) := \int_0^t \int_{\mathbb{R}} G_\lambda(t-s, x-y) \sigma\big(s, y, u(s,y)\big)\, W(\mathrm{d}y, \mathrm{d}s),$$

$$J_\lambda^4(t,x) := \int_0^t \int_{\mathbb{R}} G_\lambda(t-s, x-y) \sigma\big(s, y, u(s,y)\big) \psi(s,y)\,\mathrm{d}y\,\mathrm{d}s,$$

$$J_\lambda^5(t,x) := \int_0^{t+} \int_{\mathbb{R}} \int_{\mathrm{E}_2} G_\lambda(t-s, x-y) \sigma\big(s, y, u(s,y)\big) h(s,y,z)\, M(\mathrm{d}z, \mathrm{d}y, \mathrm{d}s).$$

In the following, we will prove Theorem 3.1.

Proof of Theorem 3.1. We now check that $S(u) \in \mathcal{H}$, if $u \in \mathcal{H}$. Applying (b) and (e) of Lemma 2.1 and Young's inequality to conclude that

$$\begin{aligned}
\big\| J_\lambda^0(u)(t,\cdot)\big\|_p &= \bigg\| \int_{\mathbb{R}} G_\lambda(t, \cdot - y) u_0(y)\,\mathrm{d}y \bigg\|_p \\
&\leqslant t^{-\frac{1}{\lambda}} \bigg\| \int_{\mathbb{R}} G_\lambda\big(1, t^{-\frac{1}{\lambda}}(\cdot - y)\big) u_0(y)\,\mathrm{d}y \bigg\|_p \\
&\leqslant t^{-\frac{1}{\lambda}} \big\| \big[G_\lambda\big(1, t^{-\frac{1}{\lambda}}\cdot\big) * u_0(\cdot)\big](\cdot)\big\|_p \\
&\leqslant t^{-\frac{1}{\lambda}} \big\| G_\lambda\big(1, t^{-\frac{1}{\lambda}}\cdot\big)\big\|_1 \big\|u_0(\cdot)\big\|_p \\
&\leqslant C\big\|u_0(\cdot)\big\|_p \\
&< \infty,
\end{aligned} \tag{3.7}$$

which is due to the fact $\mathbb{E}[\|u_0(\cdot)\|_p^p] < \infty$. We turn to $J_\lambda^1(u)$. From (a) of Lemma 3.1 with $1/r = 1/p - 1/p + 1 = 1$ and the condition (i), it follows that

$$\begin{aligned}
\mathbb{E}\big[\big\| J_\lambda^1(u)(t,\cdot)\big\|_p^p\big] &\leqslant C\mathbb{E}\Bigg[\int_0^t (t-s)^{-\frac{1}{\lambda}(1-\frac{1}{r})} \big\| f\big(s,\cdot,u(s,\cdot)\big)\big\|_p\,\mathrm{d}s\Bigg]^p \\
&\leqslant C\mathbb{E}\Bigg[\int_0^t 1 + \big\|u(s,\cdot)\big\|_p\,\mathrm{d}s\Bigg]^p \\
&\leqslant C_{p,T}\Big[1 + \sup_{0\leqslant t\leqslant T} \mathbb{E}\big[\big\|u(t,\cdot)\big\|_p^p\big]\Big] \\
&= C_{p,T}\big[1 + \big\|u(\cdot)\big\|_{\mathcal{H}}^p\big] \\
&< \infty,
\end{aligned} \tag{3.8}$$

since $u \in \mathcal{H}$. By (b) of Lemma 3.1 with $1/r = 2/p - 2/p + 1 = 1$ and $\rho = p$, together with Burkhölder inequality and the condition (i), one gets for $p \in [2, \infty[$,

$$\begin{aligned}
\mathbb{E}\big[\big\| J_\lambda^2(u)(t,\cdot)\big\|_p^p\big] &= \int_{\mathbb{R}} \mathbb{E}\big[\big|J_\lambda^2(u)(t,x)\big|^p\big]\,\mathrm{d}x \\
&\leqslant C\int_{\mathbb{R}} \mathbb{E}\Bigg[\int_0^t \int_{\mathbb{R}} G_\lambda^2(t-s, x-y)\sigma^2\big(s,y,u(s,y)\big)\,\mathrm{d}y\,\mathrm{d}s\Bigg]^{\frac{p}{2}}\,\mathrm{d}x
\end{aligned}$$

$$
\begin{aligned}
&= C\mathbb{E}\left[\left\| \int_0^t \int_{\mathbb{R}} G_\lambda^2(t-s,x-y)\sigma^2\big(s,y,u(s,y)\big)\,\mathrm{d}y\,\mathrm{d}s\right\|_{\frac{p}{2}}^{\frac{p}{2}}\right] \\
&\leqslant C\mathbb{E}\left[\int_0^t (t-s)^{-\frac{1}{\lambda}(1-r)}\big\|\sigma^2\big(s,\cdot,u(s,\cdot)\big)\big\|_{\frac{p}{2}}\,\mathrm{d}s\right]^{\frac{p}{2}} \\
&\leqslant C\mathbb{E}\left[\int_0^t (t-s)^{-\frac{1}{\lambda}(1-r)}\left(1+\big\|\big|u(s,\cdot)\big|^2\big\|_{\frac{p}{2}}\right)\mathrm{d}s\right]^{\frac{p}{2}} \\
&\leqslant C_{p,T}\Big[1+\sup_{0\leqslant t\leqslant T}\mathbb{E}\big[\big\|u(t,\cdot)\big\|_p^p\big]\Big] \\
&= C_{p,T}\big[1+\big\|u(\cdot)\big\|_{\mathcal{H}}^p\big] \\
&< \infty. \qquad (3.9)
\end{aligned}
$$

Similarly, from (a) of Lemma 3.1 with $1/r = 1/p - 2/p + 1 = 1 - 1/p \in\,]0,1]$ and assumption (3.2), it follows that for $p \in [2,\infty[$,

$$
\begin{aligned}
\mathbb{E}\big[\big\|J_\lambda^3(u)(t,\cdot)\big\|_p^p\big] &\leqslant C\mathbb{E}\left[\int_0^t (t-s)^{-\frac{1}{\lambda}(1-\frac{1}{r})}\big\|1+\big|u(s,\cdot)\big|\big\|_p\big\|\psi(s,\cdot)\big\|_p\,\mathrm{d}s\right]^p \\
&\leqslant C_p\big[1+\big\|u(\cdot)\big\|_{\mathcal{H}}^p\big]\sup_{0\leqslant t\leqslant T}\big\|\psi(t,\cdot)\big\|_p^p\left[\int_0^T (t-s)^{-\frac{1}{\lambda}(1-\frac{1}{r})(\frac{p}{p-1})}\,\mathrm{d}s\right]^{p-1} \\
&= C_{p,T}\sup_{0\leqslant t\leqslant T}\big\|\psi(t,\cdot)\big\|_p^p\big(1+\big\|u(\cdot)\big\|_{\mathcal{H}}^p\big) \\
&< \infty. \qquad (3.10)
\end{aligned}
$$

Finally, we estimate $J_\lambda^4(u)$. This is a key step in the proof of Theorem 3.1. From the condition (3.3), (b) of Lemma 3.1 with $1/r = 2/p - 4/p + 1 = 1 - 2/p \in\,]0,1]$ and Proposition 2.1, it follows that for $p \in\,]\frac{2(1+\lambda)}{\lambda-1},\infty[$,

$$
\begin{aligned}
\mathbb{E}\big[\big\|J_\lambda^4(u)(t,\cdot)\big\|_p^p\big] &= \mathbb{E}\left[\left\|\int_0^{t+}\int_{\mathbb{R}}\int_{E_2} G_\lambda(t-s,\cdot-y)\sigma\big(s,y,u(s,y)\big)h(s,y,z)\,M(\mathrm{d}y,\mathrm{d}z,\mathrm{d}s)\right\|_p^p\right] \\
&\leqslant C_p\int_{\mathbb{R}}\left[\int_0^t\int_{\mathbb{R}}\int_{E_2}\big|G_\lambda(t-s,x-y)h(s,y,z)\big|^2\big(\mathbb{E}\big[1+\big|u(s,y)\big|\big]^p\big)^{\frac{2}{p}}\mu_2(\mathrm{d}z)\,\mathrm{d}y\,\mathrm{d}s\right]^{\frac{p}{2}}\mathrm{d}x \\
&= C_p\left\|\int_0^t\int_{\mathbb{R}} G_\lambda^2(t-s,\cdot-y)\left[\int_{E_2}\big|h(s,y,z)\big|^2\mu_2(\mathrm{d}z)\right]\big(1+\mathbb{E}\big[\big|u(s,y)\big|^p\big]\big)^{\frac{2}{p}}\,\mathrm{d}y\,\mathrm{d}s\right\|_{\frac{p}{2}}^{\frac{p}{2}} \\
&\leqslant C_p\left[\int_0^t (t-s)^{-\frac{1}{\lambda}(2-r)}\left\|\left[\int_{E_2}\big|h(s,\cdot,z)\big|^2\mu_2(\mathrm{d}z)\right]\big(1+\mathbb{E}\big[\big|u(s,\cdot)\big|^p\big]\big)^{\frac{2}{p}}\right\|_{\frac{p}{4}}\mathrm{d}s\right]^{\frac{p}{2}} \\
&\leqslant C_p\left[\int_0^t (t-s)^{-\frac{p+2}{\lambda p}}\left\|\int_{E_2}\big|h(s,\cdot,z)\big|^2\mu_2(\mathrm{d}z)\right\|_{\frac{p}{2}}\big\|\big(1+\mathbb{E}\big[\big|u(s,\cdot)\big|^p\big]\big)^{\frac{2}{p}}\big\|_{\frac{p}{2}}\,\mathrm{d}s\right]^{\frac{p}{2}} \\
&\leqslant C_p\left[\sup_{0\leqslant t\leqslant T}\left\|\int_{E_2}\big|h(t,\cdot,z)\big|^2\mu_2(\mathrm{d}z)\right\|_{\frac{p}{2}}^{\frac{p}{2}}\right]\left[\sup_{0\leqslant t\leqslant T}\big\|\big(1+\mathbb{E}\big[\big|u(t,\cdot)\big|^p\big]\big)^{\frac{2}{p}}\big\|_{\frac{p}{2}}^{\frac{p}{2}}\right] \\
&\quad\times\left[\int_0^t (t-s)^{-\frac{p+2}{\lambda(p-2)}}\,\mathrm{d}s\right]^{\frac{p-2}{2}}
\end{aligned}
$$

$$\leqslant C_{p,T}\left[\sup_{0\leqslant t\leqslant T}\left\|\int_{E_2}\left|h(t,\cdot,z)\right|^2\mu_2(dz)\right\|_{\frac{p}{2}}^{\frac{p}{2}}\right]\left(1+\|u(\cdot)\|_{\mathcal{H}}^p\right)$$
$$<\infty. \tag{3.11}$$

Thus we prove that S defined by (3.6) is an operator from $\mathcal{H}$ to itself. On the other hand, from the similar argument as in (3.8)–(3.11), let $\eta>0$ sufficiently large, it's not difficult to check that there exists a constant $\kappa\in\,]0,1[$ such that

$$\left\|S(u)-S(v)\right\|_{\mathcal{H}}=\kappa\|u-v\|_{\mathcal{H}}, \tag{3.12}$$

with $u=(u(t,\cdot))_{0\leqslant t\leqslant T}$, $v=(v(t,\cdot))_{0\leqslant t\leqslant T}\in\mathcal{H}$ and $u_0=v_0$, which implies that the operator S is a contraction on $\mathcal{H}$. Therefore, there exists a $u\in\mathcal{H}$ to solve Eq. (1.1). □

Remark 3.1. Here we restrict $p>\frac{2(\lambda+1)}{\lambda-1}$, which depends on the value of $\lambda\in(1,2]$. The case when $p=2$, all the results are also right and similar as results in [22]. But it is not true for the case when $p\in(2,\frac{2(\lambda+1)}{\lambda-1}]$, because we will use a new B-D-G inequality in Proposition 2.1 to estimate $J_\lambda^4(u)$, there two places that need $p>\frac{2(\lambda+1)}{\lambda-1}$ with $\lambda\in(1,2]$. One is the use (b) of Lemma 3.1, and the other is to guarantee the integrability of $\int_0^t(t-s)^{-\frac{p+2}{\lambda(p-2)}}\,ds$.

4. Flow property of the solution

Based on Theorem 3.1, this section is devoted to studying the flow property of the global mild solution to Eq. (1.1). Let $\phi\in L^p(\mathbb{R})$, for $p\in\,]\frac{2(1+\lambda)}{\lambda-1},\infty[$ with $\lambda\in\,]1,2]$. For $(s,t,x)\in[0,T]\times[0,T]\times\mathbb{R}$, define

$$\begin{aligned}\Psi_{s,t}(\phi)(x)=&\int_{\mathbb{R}}G_\lambda(t,x-y)\phi(y)\,dy+\int_s^t\int_{\mathbb{R}}G_\lambda(t-r,x-y)f\big(r,y,u(r,y)\big)\,dy\,dr\\&+\int_s^t\int_{\mathbb{R}}G_\lambda(t-r,x-y)\sigma\big(r,y,u(r,y)\big)\,W(dy,dr)\\&+\int_s^t\int_{\mathbb{R}}G_\lambda(t-r,x-y)\sigma\big(r,y,u(r,y)\big)\psi(r,y)\,dy\,dr\\&+\int_s^{t+}\int_{\mathbb{R}}\int_{E_2}G_\lambda(t-s,x-y)\sigma\big(r,y,u(r,y)\big)h(r,y,z)\,M(dz,dy,dr).\end{aligned}\tag{4.1}$$

Suppose that the assumptions of Theorem 3.1 hold, then Theorem 3.1 yields that for any initial state (s,x), there exists a unique solution $\Psi_{s,t}(\phi(x))$ starting from x at time s, and for each pair $(s,t)\in[0,T]\times[0,T]$ and $p\in\,]\frac{2(1+\lambda)}{\lambda-1},\infty[$,

$$\Psi_{s,t}:L^p(\mathbb{R})\to L^p(\mathbb{R}),\quad \mathbb{P}\text{-a.s.}$$

Thus like the case of the solution of an ordinary differential equation, it is reasonable to ask whether the solution $\Psi_{s,t}(\phi(x))$ satisfies some flow property (see, e.g. Fujiwara and Kunita [12,13]). Fortunately, we have the following flow property of the global mild solution to Eq. (1.1).

Theorem 4.1. *Under the assumptions of Theorem* 3.1:

(i) *For each* $T>0$,

$$\Psi_{t,t}=I,\quad \forall 0\leqslant t\leqslant T,\ \mathbb{P}\text{-a.s.}$$

(ii) *For each* $T>0$,

$$\Psi_{s',t}\circ\Psi_{s,s'}=\Psi_{s,t},\quad \forall 0\leqslant s\leqslant s'\leqslant t\leqslant T,\ \mathbb{P}\text{-a.s.}$$

Proof. We first prove (i). Note that $G_\lambda(0,x)=\delta(x)$ for $x\in\mathbb{R}$. By (4.1), the operator $\Psi_{s,t}$ is clearly an identity I, if the last term of right-hand side of (4.1) equals zero with $s=t$ almost surely under $\mathbb{P}$. Indeed,

$$\begin{aligned}
&\mathbb{E}\left[\left|\int_t^{t+}\int_{\mathbb{R}}\int_{\mathrm{E}_2}\delta_0(x-y)\sigma\big(r,y,u(r-,y)\big)h(r,y,z)\,M(\mathrm{d}z,\mathrm{d}y,\mathrm{d}r)\right|^2\right]\\
&=\int_t^{t+}\int_{\mathbb{R}}\int_{\mathrm{E}_2}\delta_0(x-y)\mathbb{E}\big[\sigma^2\big(r,y,u(r-,y)\big)h^2(r,y,z)\big]\,\mu_2(\mathrm{d}z)\,\mathrm{d}y\,\mathrm{d}r\\
&=\int_t^{t+}\left[\int_{\mathrm{E}_2}\mathbb{E}\big[\sigma^2\big(r,x,u(r,x)\big)h^2(r,x,z)\big]\,\mu_2(\mathrm{d}z)\right]\mathrm{d}r\\
&=0.
\end{aligned}\tag{4.2}$$

Next we prove (ii). Note that for $\phi\in L^p(\Omega)$ with $p\in\,]\frac{2(1+\lambda)}{\lambda-1},\infty[$,

$$\begin{aligned}
&\int_{\mathbb{R}} G_\lambda\big(t-s',x-y\big)\big[\Psi_{s,s'}(\phi)\big](y)\,\mathrm{d}y\\
&\quad=\int_{\mathbb{R}} G_\lambda\big(t-s',x-y\big)\left[\int_{\mathbb{R}} G_\lambda\big(s'-r,y-\xi\big)\phi(\xi)\,\mathrm{d}\xi\right]\mathrm{d}x\\
&\qquad+\int_{\mathbb{R}} G_\lambda\big(t-s',x-y\big)\left[\int_s^{s'}\int_{\mathbb{R}} G_\lambda\big(s'-r,y-\xi\big)f\big(r,\xi,u(r,\xi)\big)\,\mathrm{d}\xi\,\mathrm{d}r\right]\mathrm{d}y\\
&\qquad+\int_{\mathbb{R}} G_\lambda\big(t-s',x-y\big)\left[\int_s^{s'}\int_{\mathbb{R}} G_\lambda\big(s'-r,y-\xi\big)\sigma\big(r,\xi,u(r,\xi)\big)\,W(\mathrm{d}\xi,\mathrm{d}r)\right]\mathrm{d}y\\
&\qquad+\int_{\mathbb{R}} G_\lambda\big(t-s',x-y\big)\left[\int_s^{s'}\int_{\mathbb{R}} G_\lambda\big(s'-r,y-\xi\big)\sigma\big(r,\xi,u(r,\xi)\big)\psi(r,\xi)\,\mathrm{d}\xi\,\mathrm{d}r\right]\mathrm{d}y+\int_{\mathbb{R}} G_\lambda\big(t-s',x-y\big)\\
&\qquad\times\left[\int_s^{s'+}\int_{\mathbb{R}} G_\lambda\big(s'-r,y-\xi\big)\sigma\big(r,\xi,u(r,\xi)\big)h(r,\xi,z)\,M(\mathrm{d}z,\mathrm{d}\xi,\mathrm{d}r)\right]\mathrm{d}y.
\end{aligned}$$

Thus by a generalized Fubini theorem (see e.g. Proposition 2.2 in [21]) and (d) of Lemma 2.1,

$$\begin{aligned}
\int_{\mathbb{R}} G_\lambda\big(t-s',x,y\big)\big[\Psi_{s,s'}(\phi)\big](y)\,\mathrm{d}y&=\int_{\mathbb{R}} G_\lambda(t-r,x-\xi)\phi(\xi)\,\mathrm{d}\xi\\
&\quad+\int_s^{s'}\int_{\mathbb{R}} G_\lambda(t-r,x-\xi)f\big(r,\xi,u(r,\xi)\big)\,\mathrm{d}\xi\,\mathrm{d}r\\
&\quad+\int_s^{s'}\int_{\mathbb{R}} G_\lambda(t-r,x-\xi)\sigma\big(r,\xi,u(r,\xi)\big)\,W(\mathrm{d}\xi,\mathrm{d}r)\\
&\quad+\int_s^{s'}\int_{\mathbb{R}} G_\lambda(t-r,x-\xi)\sigma\big(r,\xi,u(r,\xi)\big)\psi(r,\xi)\,\mathrm{d}\xi\,\mathrm{d}r\\
&\quad+\int_s^{s'+}\int_{\mathbb{R}} G_\lambda(t-r,x-\xi)\sigma\big(r,\xi,u(r,\xi)\big)h(r,\xi,z)\,M(\mathrm{d}z,\mathrm{d}\xi,\mathrm{d}r).
\end{aligned}\tag{4.3}$$

Then by (4.2), for all $\phi\in L^p(\Omega)$ with $p\in\,]\frac{2(1+\lambda)}{\lambda-1},\infty[$,

$$
\begin{aligned}
[\Psi_{s',t} \circ \Psi_{s,s'}](\phi)(x) = & \int_{\mathbb{R}} G_\lambda\big(t-s', x-y\big)\big[\Psi_{s,s'}(\phi)\big](y)\,\mathrm{d}y + \int_{s'}^{t}\int_{\mathbb{R}} G_\lambda(t-r, x-\xi) f\big(r,\xi,u(r,\xi)\big)\,\mathrm{d}\xi\,\mathrm{d}r \\
& + \int_{s'}^{t}\int_{\mathbb{R}} G_\lambda(t-r, x-\xi)\sigma\big(r,\xi,u(r,\xi)\big)\,W(\mathrm{d}\xi,\mathrm{d}r) \\
& + \int_{s'}^{t}\int_{\mathbb{R}} G_\lambda(t-r, x-\xi)\sigma\big(r,\xi,u(r,\xi)\big)\psi(r,\xi)\,\mathrm{d}\xi\,\mathrm{d}r \\
& + \int_{s'}^{t+}\int_{\mathbb{R}} G_\lambda(t-r, x-\xi)\sigma\big(r,\xi,u(r,\xi)\big)h(r,\xi,z)\,M(\mathrm{d}z,\mathrm{d}\xi,\mathrm{d}r) \\
= & \Psi_{s,t}(\phi)(x).
\end{aligned} \tag{4.4}
$$

This completes the proof of the theorem. □

5. Examples

In this section, we shall give two examples to illustrate our results derived above.

Example 5.1. Recall $\dot{W}$, $\dot{M}$, $\dot{N}$ defined in (2.4). We consider the following fractional equation:

$$
\begin{cases}
\dfrac{\partial u(t,x)}{\partial t} = \Delta_\lambda u(t,x) + u(t,x) + u(t,x)\dot{W}(t,x) + \displaystyle\int_{U_0} u(t,x)\left(\frac{1}{1+x^2}\right)^p z\,\dot{M}(\mathrm{d}z,x,t) \\
\qquad + \displaystyle\int_{\mathrm{E}_2/U_0} u(t,x)\left(\frac{1}{1+x^2}\right)^p \dot{N}(\mathrm{d}z,x,t), \\
u(0,x) = u_0(x).
\end{cases} \tag{5.1}
$$

Recall Eq. (3.1). Here let $f(t,x,u(t,x)) = u(t,x)$, $\sigma(t,x,u(t,x)) = u(t,x)$, $h_1(t,x,z) = \frac{1}{(1+x^2)^p} z$ and $h_2(t,x,z) = \frac{1}{(1+x^2)^p}$, for $p \in]\frac{2(\lambda+1)}{\lambda-1}, +\infty[$ with $\lambda \in]1,2]$. Then

$$
\begin{aligned}
\big\|\Psi(t,\cdot)\big\|_p^p &= \left(\int_{\mathbb{R}} \left|\int_{\mathrm{E}_2/U_0} h_2(t,x,z)\,\mu_2(\mathrm{d}z)\right|^p \mathrm{d}x\right)^{\frac{1}{p}} \\
&= C_p \pi^{\frac{1}{p}} \mu_2(\mathrm{E}_2/U_0) \\
&< +\infty
\end{aligned} \tag{5.2}
$$

and

$$
\begin{aligned}
\left\|\int_{\mathrm{E}_2} \big|h(t,\cdot,z)\big|^2 \mu_2(\mathrm{d}z)\right\|_{\frac{p}{2}}^{\frac{p}{2}} &= \left\|\int_{\mathrm{E}_2} \big[h_1(t,\cdot,z)\mathbf{1}_{U_0}(z) + h_2(t,\cdot,z)\mathbf{1}_{\mathrm{E}_2/U_0}(z)\big]^2 \mu_2(\mathrm{d}z)\right\|_{\frac{p}{2}}^{\frac{p}{2}} \\
&\leqslant \left(\int_{\mathbb{R}} \left(\int_{U_0} h_1^2(t,x,z)\,\mu_2(\mathrm{d}z) + \int_{\mathrm{E}_2/U_0} h_2^2(t,x,z)\,\mu_2(\mathrm{d}z)\right)^{\frac{p}{2}} \mathrm{d}x\right)^{\frac{2}{p}} \\
&\leqslant C_p \pi^{\frac{2}{p}} \left(\mu_2(\mathrm{E}_2/U_0) + \int_{U_0} z^2\,\mu_2(\mathrm{d}z)\right) \\
&< +\infty.
\end{aligned} \tag{5.3}
$$

So the assumptions (i) and (ii) of Theorem 3.1 and Theorem 4.1 hold, we obtain the main results of Theorem 3.1 and Theorem 4.1.

Example 5.2. Let us consider a fractional equation driven by a compensated Poisson noise (pure jumps) as follows:

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} = \Delta_\lambda u(t,x) + \displaystyle\int_Z u(t,x) e^{-\frac{z^2}{2}} \dot{M}(\mathrm{d}z, x, t), \\ u(0,x) = u_0(x). \end{cases} \tag{5.4}$$

Here $\sigma(t,x,u(t,x)) = u(t,x)$ and $h_1(t,x,z) = h_2(t,x,z) = e^{-\frac{z^2}{2}}$. Let $\Pi(Z) < +\infty$, then it is easy to check that (i) and (ii) of Theorem 3.1 and Theorem 4.1 holds. When $\lambda = 2$, similar results as Theorem 3.1 are proved in [1,5,16]. In this paper we could improve those results in [1,5,16] and prove flow property of the solution of Eq. (5.4).

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